

Notes on Cube Notation

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1 Introduction

It is necessary to have a method of efficiently communicating manipulation of the Rubik's Cube. Imagine if I had a cube which was scrambled such that it took exactly 4 moves to solve. My friend elsewhere has figured out how to solve this cube but must tell me in a verbal or written manner. He could say (or write) something resembling the following:

“First, turn the right face clockwise 90°. Second, turn the top face clockwise 90°. Third, turn the right face counterclockwise 90°. Last, turn the top face counterclockwise 90°.”

Sure, this communicates the point, but imagine that my friend's devised solution required 80 moves. How long would it take to tell someone how to turn the cube to solve it, and how much space would the solution take up on paper? Thus, we need a better, shorter way to describe cube moves. Note that this write-up is intended for consultation alongside the Week 1 PowerPoint—see section titles as written below.

2 Slide “3x3 Notation – Face Turns”

The Rubik's Cube has 6 faces, so we assign each face a letter according to its position facing the solver. We have “U” for Up/Top, “D” for Down/Bottom, “L” for Left, “R” for Right, “F” for Front, and “B” for Back. Each letter, by itself, indicates a 90° turn of that face in the clockwise direction.

Example: U

Adding an apostrophe behind a letter (pronounced “prime”) indicates that the turn is performed in the counterclockwise direction.

Example: U'

Adding a “2” behind a letter indicates that the 90° turn is performed twice, thus yielding an overall 180° turn.

Example: U2

It is extremely important to remember that the directionality of turns (i.e. clockwise vs counterclockwise) is determined on the assumption that the face in question is directly in front of the solver.

We will copiously use notation all semester, so the better you understand face turn conventions, the less trouble you will have when we begin discussing solution methods, especially those of the Rubik's Revenge/4x4 and other higher-order regular cuboids that require more complicated notation.

3 Slide “3x3 Notation – Wide Turns”

We have covered turns of single faces, but what if we want to turn two layers at once? The notation for these so-called wide turns is the same as the notation for regular face turns, except that the letters are now lowercase. Alternatively, we add “w” behind the analogous uppercase letter. Both are acceptable ways of indicating wide turns.

Example: u/Uw, u'/Uw', u2/Uw2

4 Slide “3x3 Notation – Rotations”

On occasion, we would like to indicate that the entire cube should be reoriented, or rotated, in a specific direction without performing any turns of individual faces/layers. The cube can be rotated along the x-, y-, and z-axes, and the names of the axes are the letters that are used to denote such rotations—“x,” “y,” and “z.”

Example: x, x', x2

There are two faces that turn along each axis. For example, the L and R faces turn along the x-axis. To which face should the directionality of a rotation match? In other words, the terms “clockwise” and “counterclockwise” can only be assigned to faces that are directly facing the solver. In the case of x rotations, should a clockwise x rotation be performed assuming that the L face is towards the solver, or assuming that the R face is towards the solver?

Rotation directionality is assigned based on proximity of the rotation letter from the corresponding two face letters in the alphabet, without looping. As a result:

x rotations are performed in the same direction as R turns, x is closer to R than L
y rotations are performed in the same direction as U turns, y is closer to U than D
z rotations are performed in the same direction as F turns, z is closer to F than B

5 Slide “3x3 Notation – Slice Turns”

The last type of notation exists for layers that are sandwiched between two other layers, also known as slice layers, which are also assigned letters. These are “M” for the Middle layer, “E” for the Equatorial layer, and “S” for the Standing layer.

Example: M, M', M2

Like rotations, the directionality of slice turns is dependent on the adjacent non-slice layer that is assumed to be facing the solver. Direction conventions are assigned using the same rule as that for rotations:

M turns are performed in the same direction as L turns, M is closer to L than R
E turns are performed in the same direction as D turns, E is closer to D than U
S turns are performed in the same direction as F turns, S is closer to F than B

6 Slide “Redundant Notation”

Cube notation is redundant. That is, there are multiple ways to express any given move of the Rubik’s Cube, and certain notation may be simplified.

Face turn redundancy – The directionality of 180° turns is irrelevant. A clockwise 180° turn will produce the same result as a counterclockwise 180° turn.

Example: U2 = U2'

Additionally, notations for 270° and 360° turns are rarely seen. That is because 270° turns produce the same result as 90° turns in the opposite direction, and 360° turns produce the same result as doing nothing at all.

Example: $U3 = U'$, $U4 = \text{no move}$

Slice turn redundancy – All slice turns can be expressed as opposite-direction turns of the two adjacent faces, followed by a rotation along the same axis.

Example: $M = L' R x'$

Wide turn redundancy – All wide turns can be expressed as turns of the opposite face, followed by a rotation along the same axis.

Example: $u/Uw = D y$

Rotation redundancy – All rotations can be expressed as opposite-direction face turns and a slice turn along that axis.

Example: $x = R M' L'$